Lec 9

state space

→ The system is described it's dynamics in time domain

$$\boxed{\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) + D\,u(t) \end{aligned}}$$

Two state equation

Where $u(t) \rightarrow$ i/P

$y(t) \rightarrow$ o/P

درجة البسط في ال T.F أقل من درجة المقام

$(D = 0) \rightarrow$ (Physical {system) 1 ⇐

$n \rightarrow$ system order

$A_{n*n} \Rightarrow$ system matrix

$B_{n*1} \Rightarrow$ i/P matrix

$C_{1*n} \Rightarrow$ o/P matrix ($x(t)_{n\times 1} \Rightarrow$ state vector

$$x(t)_{n*1} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

(measuring values)

$x_1(t), x_2(t), \dots, x_n(t) \rightarrow$ states of the system.

~~Control Lec~~

~~state space~~

system

- T.F (differential eqns)
- Block diagram or signal flow graph
- two state eqn.

$$\boxed{\begin{aligned}\dot{x} &= Ax + Bu \\ y &= cx + Du\end{aligned}}$$

- Physical model

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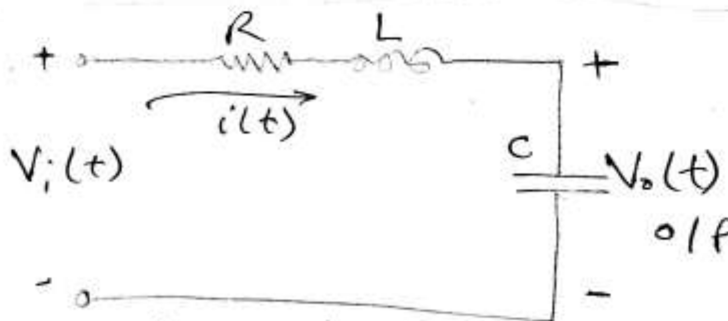
*

$i(t) = x(t)$

$V_o(t) = X_2(t)$



$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \\ \end{pmatrix} V_i(t)$$

$$y(t) = \begin{pmatrix} & \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

$$V_i(t) = V_R + V_L + V_o$$

$$= \underbrace{i(t)}_{x_1(t)} R + L \underbrace{\frac{di(t)}{dt}}_{\dot{x}_1(t)} + \underbrace{V_o(t)}_{x_2(t)}$$

$$L\dot{x}_1(t) = -R\,x_1(t) - x_2(t) + V_i(t)$$

$$\boxed{\dot{x}_1(t) = -\frac{R}{L}\,x_1(t) - \frac{1}{L}\,x_2(t) + \frac{1}{L}\,V_i(t)}$$

$$\underbrace{i(t)}_{x_1(t)} = C \underbrace{\frac{dV_o(t)}{dt}}_{\dot{x}_2(t)}$$

$$\boxed{\dot{x}_2(t) = \frac{1}{C}\,x_1(t)}$$

$$V_o(t) = x_2(t) \Rightarrow y(t) = x_2(t)$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{L} & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} V_i(t)$$

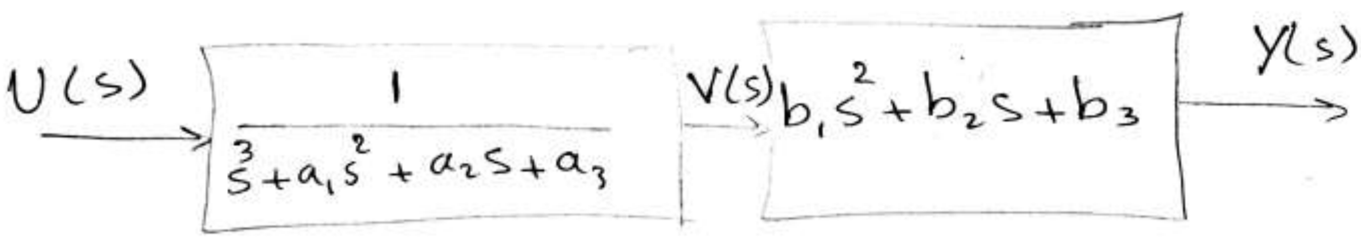
$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

Canonical Forms for state space

1 Controllable form

<u>3rd order system</u>

$$T.F = \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$



<u>assume</u>

$V = x_1$

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \longrightarrow 1 \\ \dot{x}_2 &= x_3(t) \longrightarrow 2 \\ \dot{x}_3 &= \text{----} \end{aligned}$$

$$\frac{V(s)}{U(s)} = \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$(s^3 + a_1 s^2 + a_2 s + a_3) V(s) = U(s) \Big\downarrow \mathcal{L}^{-1}.T$$

$$\underbrace{\dddot{v}_1(t)}_{\dot{x}_3} + a_1 \underbrace{\ddot{v}(t)}_{x_3} + a_2 \underbrace{\dot{v}(t)}_{x_2} + a_3 \underbrace{v(t)}_{x_1} = u(t)$$

$\cancel{\dot{x}_3 + a_1}$

$$\boxed{\dot{x}_3 = -a_3 x_1 - a_2 x_2(t) - a_1 x_3 + u(t)} \rightarrow (3)$$

~~...~~ 1, 2, 3 $\rightarrow$ first state eqn.

$$\frac{Y(s)}{V(s)} = b_1 s^2 + b_2 s + b_3$$

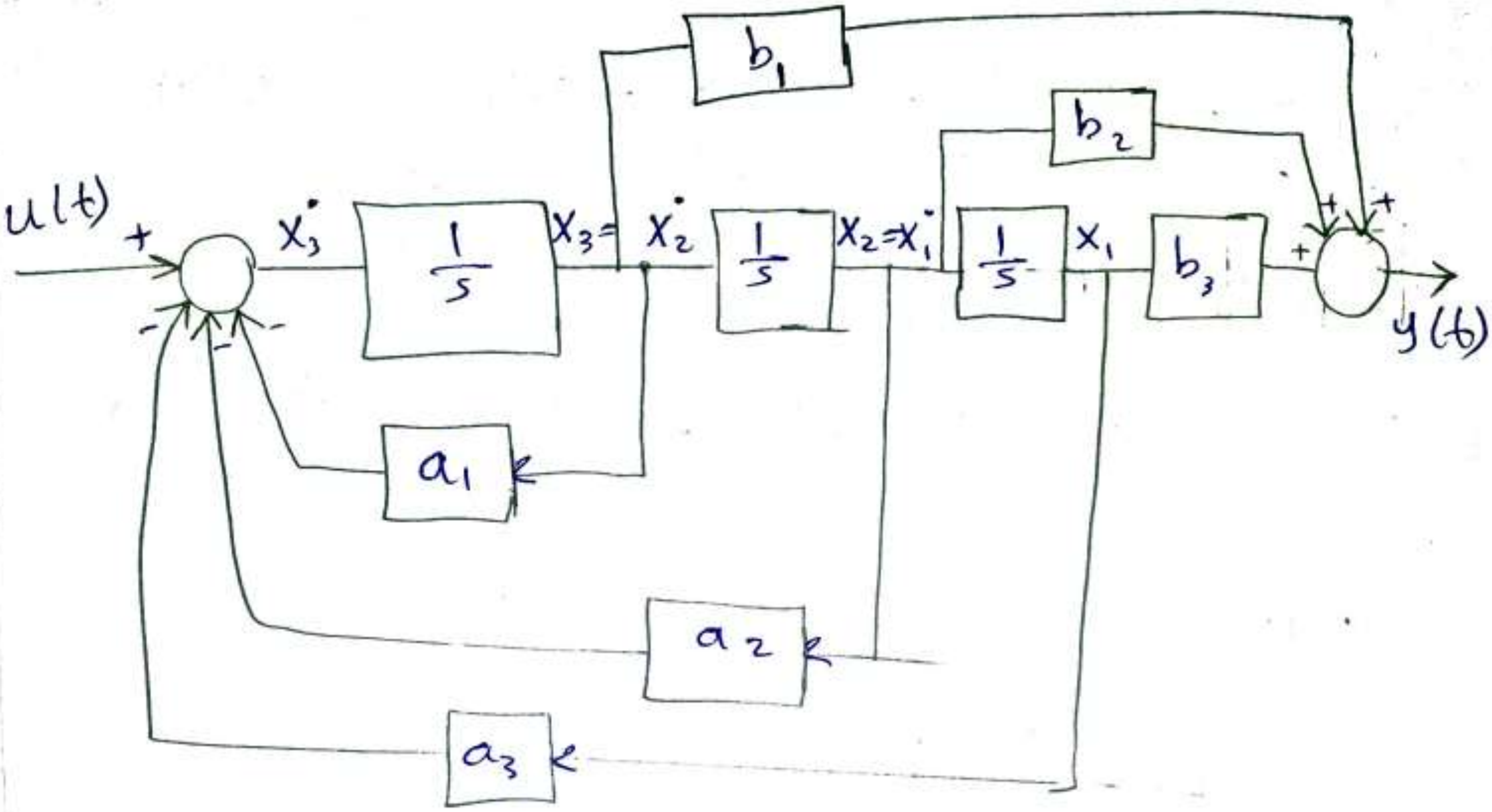
$$Y(s) = \left(b_1 s^2 + b_2 s + b_3\right) V(s)$$

$\mathcal{L}^{-1}.T \Downarrow$

$$y(t) = b_1 \underbrace{\ddot{v}(t)}_{x_3} + b_2 \underbrace{\dot{v}(t)}_{x_2} + b_3 \underbrace{v(t)}_{x_1}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} b_3 & b_2 & b_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$



مـ طالما إني قادر أوصل الـ i/p لأي (state)
موجوده عندنا يبقى النظام (Controllable)

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

first $\rightarrow$ check that Coeff. of $s^3 = 1$
أكبر أس

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

دي معاملات المقام
معكوسة

معامل s^3

$$y(t) = \underbrace{\begin{pmatrix} b_3 & b_2 & b_1 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

معاملات البسط معكوسة
لكن بنفس الإشارة.

$$T.F = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} b_4 & b_3 & b_2 & b_1 \end{pmatrix} x(t)$$

<u>2nd order</u> $\quad T.F = \dfrac{b_1 s + b_2}{s^2 + a_1 s + a_2}$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} b_2 & b_1 \end{pmatrix} x(t)$$

2 observable form

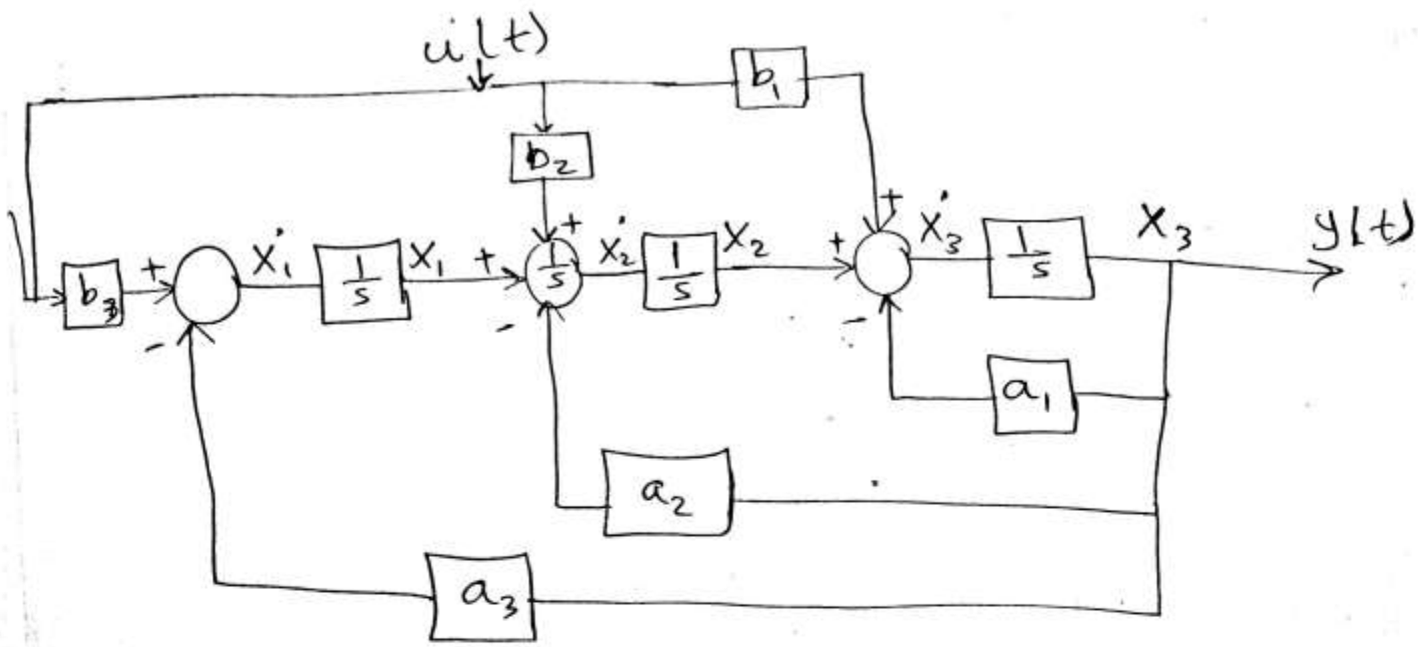
$$A_o = A_c^T \qquad B_o = B_c^T \qquad C_o = C_c^T$$

3rd order system

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} b_3 \\ b_2 \\ b_1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$



4th order system

$$T.F = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & -a_4 \\ 1 & 0 & 0 & -a_3 \\ 0 & 1 & 0 & -a_2 \\ 0 & 0 & 1 & -a_1 \end{bmatrix} x(t) + \begin{bmatrix} b_4 \\ b_3 \\ b_2 \\ b_1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t)$$

مصفوفة مكونة من

$\Rightarrow x_1, x_2, x_3, x_4$

EX

$$T.F = \frac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)}$$

* Find the state-space model in Controllable and observable forms.
* Draw the state diagram for each case

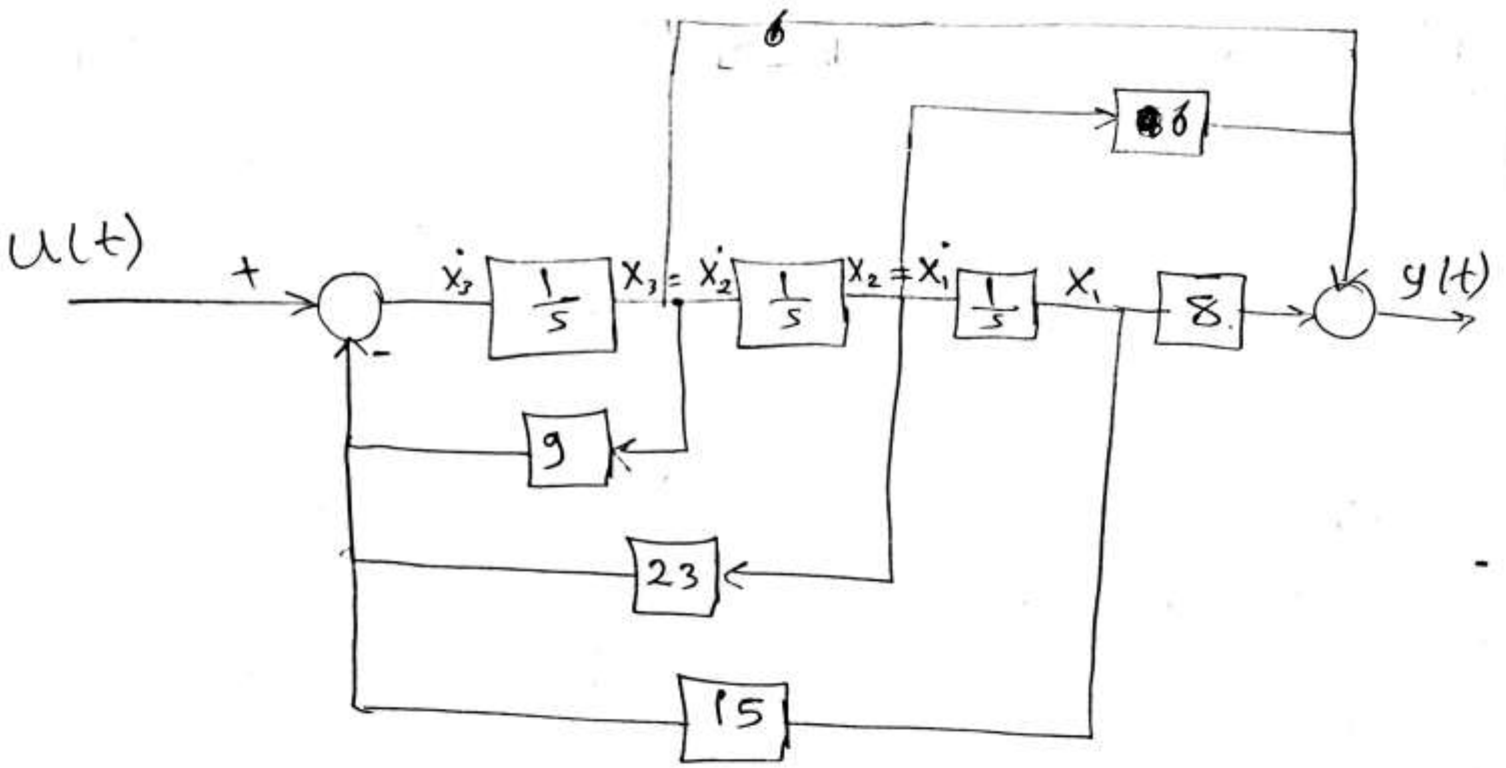
Sol

$$T.F = \frac{s^2 + 6s + 8}{s^3 + 9s^2 + 23s + 15}$$

[1] Controllable form

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

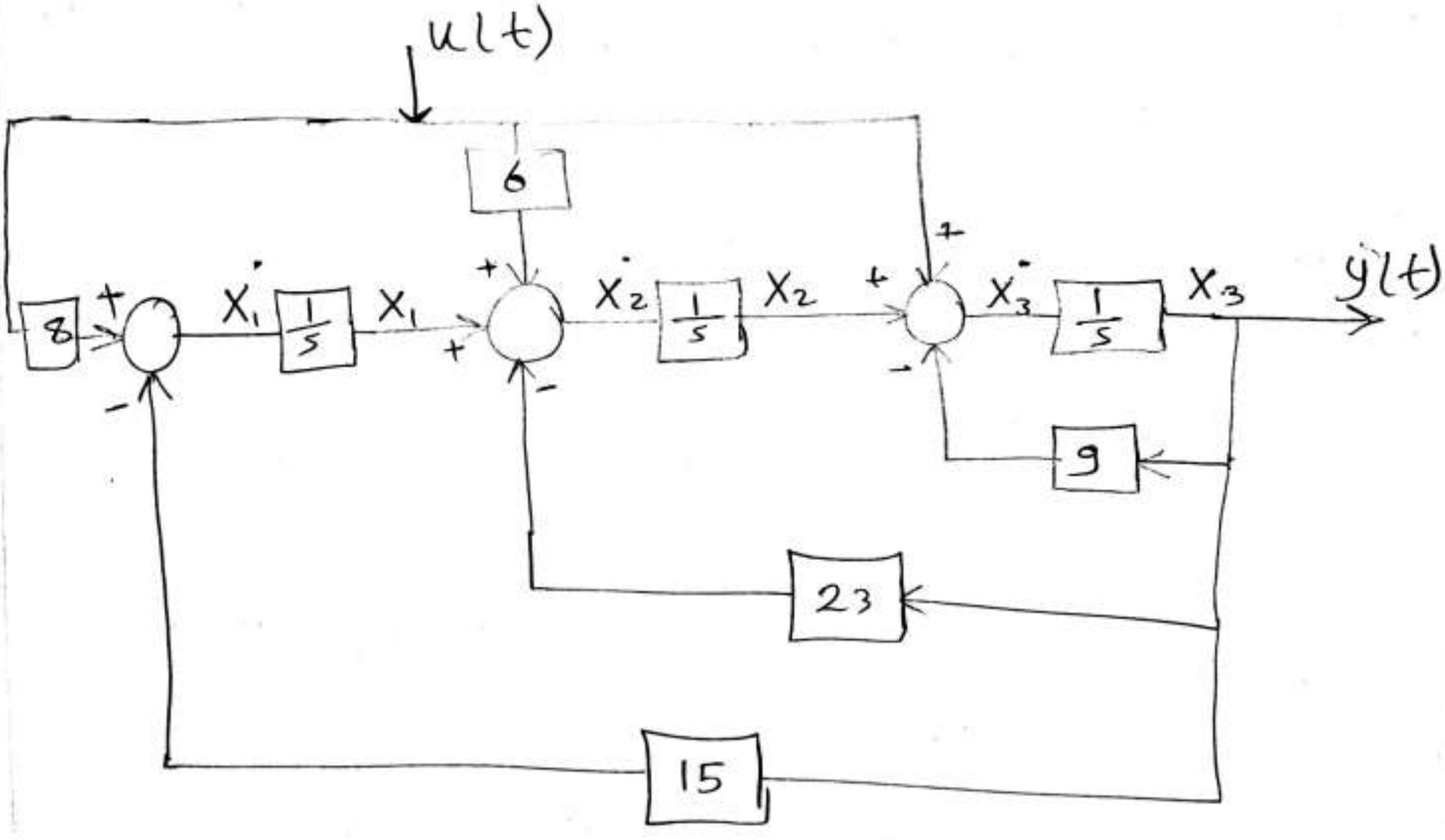
$$y(t) = \begin{pmatrix} 8 & 6 & 1 \end{pmatrix} x(t)$$



2 observable

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & -15 \\ 1 & 0 & -23 \\ 0 & 1 & -9 \end{bmatrix} x(t) + \begin{pmatrix} 8 \\ 6 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t)$$



3 Diagonal Form

3rd order system.

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{(s+P_1)(s+P_2)(s+P_3)} = \frac{A_1}{(s+P_1)} + \frac{A_2}{(s+P_2)} + \frac{A_3}{(s+P_3)}$$

$$Y(s) = \underbrace{\frac{U(s) \cdot A_1}{(s+P_1)}}_{X_1(s)} + \underbrace{\frac{U(s) \cdot A_2}{(s+P_2)}}_{X_2(s)} + \underbrace{\frac{U(s) \cdot A_3}{(s+P_3)}}_{X_3(s)}$$

$$X_1(s) = \frac{U(s)}{s+P_1} \Rightarrow U(s) = (s+P_1)\, X_1(s)$$

$$u(t) = \dot{x}_1(t) + P_1\, x(t)$$

$$\boxed{\dot{x}_1 = -P_1\, x_1 + u(t)}$$

$$\boxed{\begin{aligned}\dot{x}_2(t) &= -P_2 x_2 + u(t)\\ \dot{x}_3(t) &= -P_3 x_3 + u(t)\end{aligned}}$$

$$\dot{x}(t) = \begin{bmatrix} -P_1 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -P_3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$Y(s) = A_1 X_1(s) + A_2 X_2(s) + A_3 X_3(s)$$

$$\boxed{y(t) = A_1 x_1(t) + A_2 x_2(t) + A_3 x_3(t)}$$

$$y(t) = \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} x(t)$$

$$T.F = \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$= \frac{b_1 s^2 + b_2 s + b_3}{(s+P_1)(s+P_2)(s+P_3)} = \frac{A_1}{s+P_1} + \frac{A_2}{s+P_2} + \frac{A_3}{s+P_3}$$

3 Poles $\Rightarrow -P_1, -P_2, -P_3$.

$$\dot{X}(t) = \begin{pmatrix} -P_1 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -P_3 \end{pmatrix} X(t) + \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\text{كل عناصرها تساوي 1}} u(t)$$

لب القطر الرئيسي عبارة عن ال Poles

$$y(t) = \underbrace{\begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix}}_{\text{عناصر البسط}} X(t)$$

<u>for repeated Poles</u>

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{(s+P_1)^2 (s+P_2)}$$

Poles $\Rightarrow -P_1, -P_1, -P_2$

$$Y(s) = \frac{U(s)\, A_1}{(s+P_1)^2} + \frac{U(s)\, A_2}{s+P_1} + \frac{U(s)\, A_3}{s+P_2}$$

$$X_1(s) = \frac{U(s)}{(s+P_1)^2} = \frac{U(s)}{s+P_1} \cdot \frac{1}{s+P_1}$$

$$X_1(s) = \frac{X_2(s)}{s+P_1} \rightarrow \boxed{\dot{x}_1 = -P_1 x_1 + x_2}$$

$$X_2(s) = \frac{U(s)}{s+P_1} \rightarrow \dot{x}_2 = -P_1 x_2 + u(t)$$

$$\dot{x}_3 = -P_2 x_3 + u(t)$$

$$\dot{x}(t) = \begin{pmatrix} -P_1 & 1 & 0 \\ 0 & -P_1 & 0 \\ 0 & 0 & -P_2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

Ex : $T.F = \frac{s^2 + 6x + 8}{(s+1)(s+3)(s+5)}$

- find the state space mode in Diagonal form
- Draw the state diagram

Sol

$T.f = \frac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)}$ | using P.F

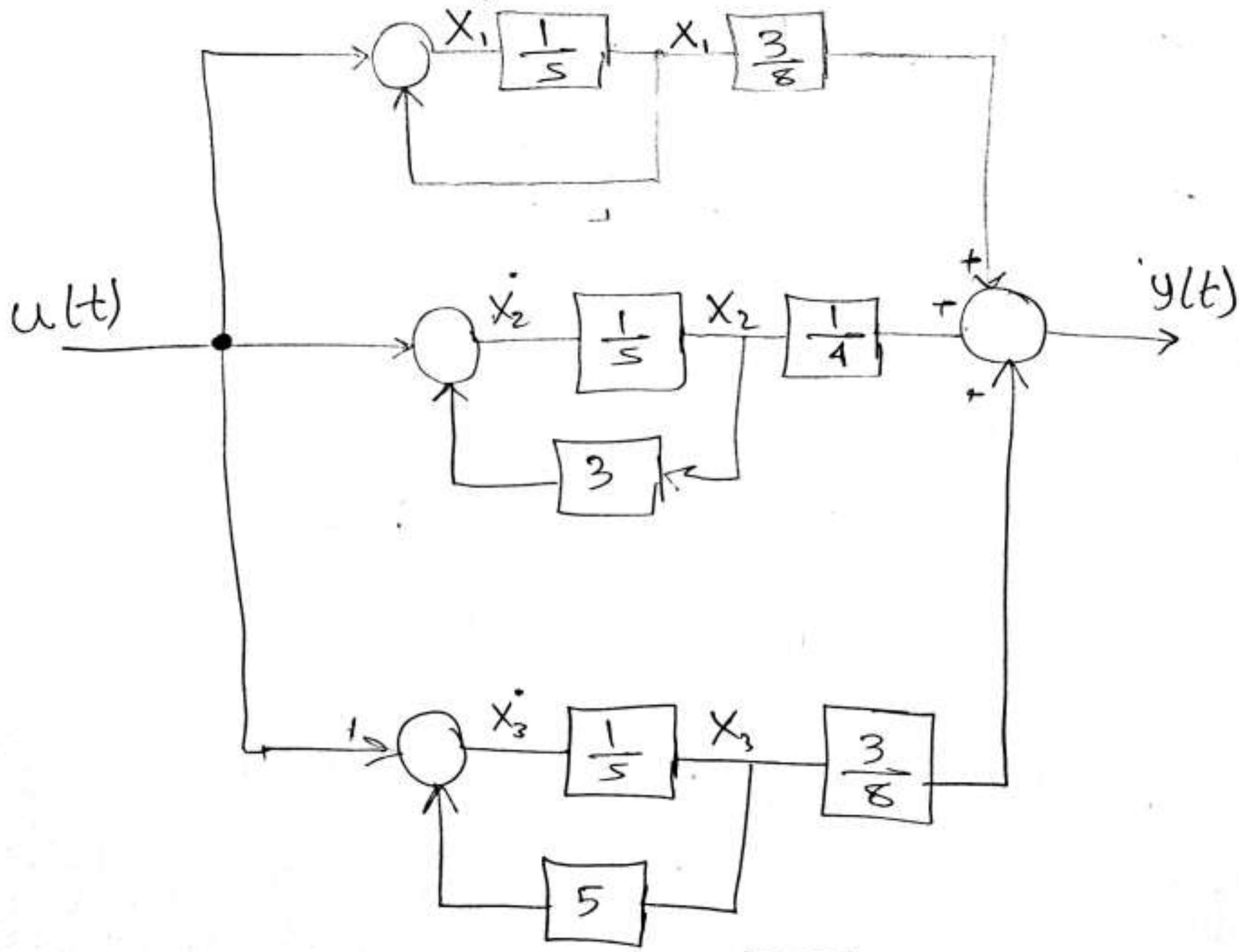
$$= \frac{A_1}{(s+1)} + \frac{A_2}{(s+3)} + \frac{A_3}{(s+5)}$$

$A_1 = \frac{1 - 6 + 8}{(2)(4)} = \frac{3}{8}$ | $A_2 = \frac{9 - 18 + 8}{(-2)(2)} = \frac{-1}{-4} = \frac{1}{4}$

$A_3 = \frac{25 - 30 + 8}{(-4)(-2)} = \frac{3}{8}$

$$\dot{x}(t) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} \frac{3}{8} & \frac{1}{4} & \frac{3}{8} \end{pmatrix} x(t)$$



$\boxed{Ex}$

$$T.F = \frac{2s^2+6s+5}{(s+1)^2(s+2)} \Downarrow P.F$$

$$T.F = \frac{A_1}{(s+1)^2} + \frac{A_2}{s+1} + \frac{A_3}{s+2}$$

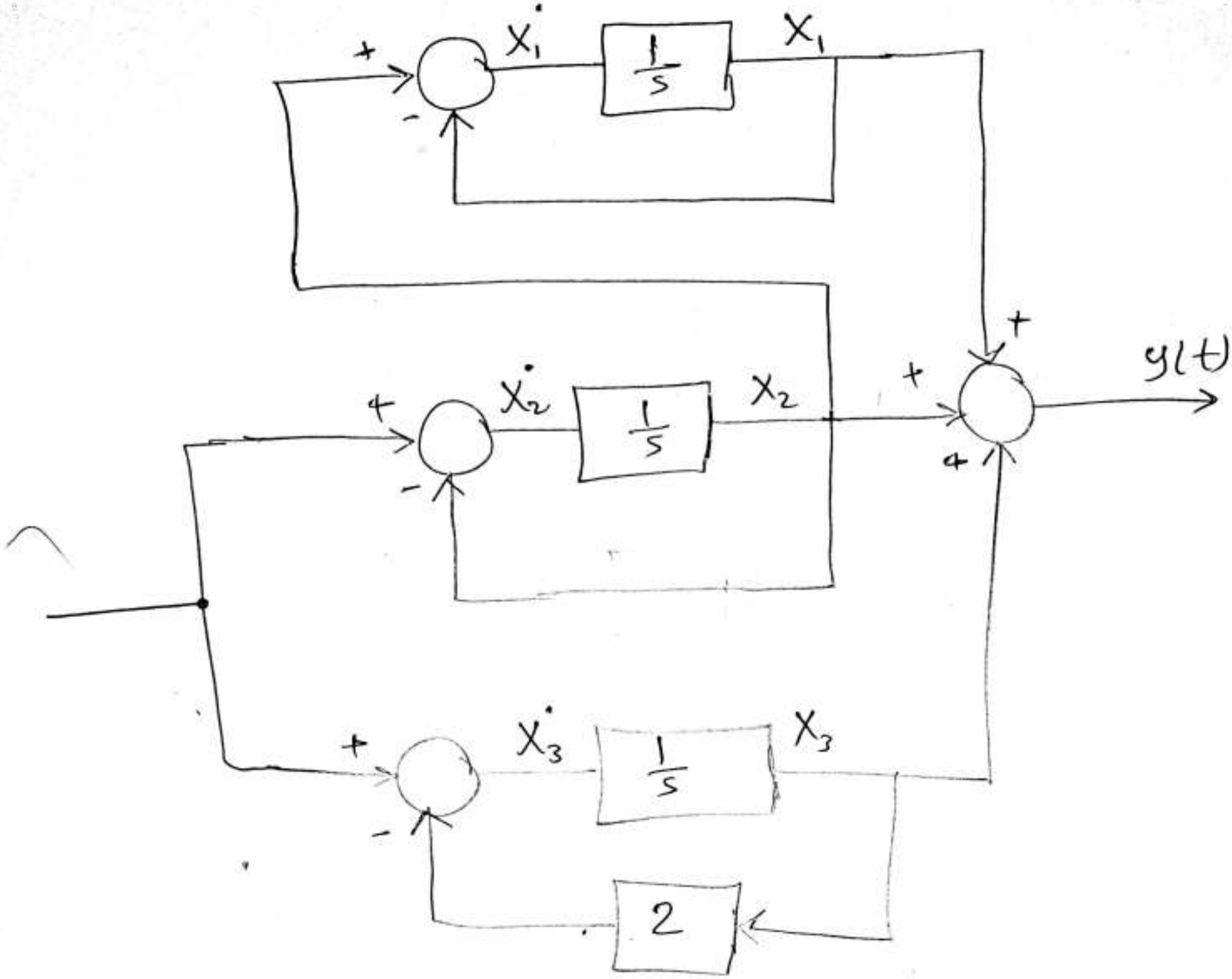
$$A_1 = \frac{2-6+5}{-1+2} = 1 \qquad , A_3 = \frac{8-12+5}{1} = 1$$

Put $s=0$ $\quad \frac{5}{2} = A_1 + A_2 + \frac{A_3}{2}$

$\hookrightarrow A_2 = 1$

$$\dot{x}(t) = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} x(t)$$



state-space Analysis

given the two state equations

$$\boxed{\begin{aligned} \dot{x} &= A x + B u \\ y &= c x \end{aligned}}$$

[1] T.F

$$\boxed{\begin{aligned} sX(s) &= A\,X(s) + B\,u(s) \longrightarrow (1) \\ Y(s) &= C\,X(s) \longrightarrow (2) \end{aligned}}$$

$$sX(s) - A\,X(s) = B\,u(s)$$

$$\underset{\downarrow}{(sI - A)}\,X(s) = B\,U(s)$$

sI

S مضروبة في
مصفوفة الوحدة

$I_{n*n} \rightarrow$ identity matrix

$$X(s) = (sI - A)^{-1}\,B\,u(s) \longrightarrow (3)$$

from (3) in (2)

$$Y(s) = C\,(sI - A)^{-1}\,B\,U(s)$$

$$\boxed{T.F = \frac{Y(s)}{U(s)} = C\,(sI - A)^{-1}\,B}$$

2 ch. equation

$$|sI - A| = 0$$

ـه تنتج معادلة جذورها هيا ال (Poles) بتاعة ال (system).

$\hookrightarrow$ roots of ch. equation $\equiv$ Poles $\equiv$ eigen values.

3 The system $\underbrace{\text{response}}_{\text{o/P in time domain.}}$ to i/P $u(t)$

if $x(0) \neq 0$

$$\dot{x}(t) = A\,x(t) + B\,u(t) \xrightarrow{L.T}$$

$$s\,X(s) = A\,X(s) + B\,U(s)$$

$$s\,X(s) - A\,X(s) = x(0) + B\,U(s)$$

$$(sI - A)\,X(s) = x(0) + B\,U(s)$$

$$X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} B\,U(s)$$

Let: $\phi(s) = (sI - A)^{-1} \Rightarrow$ transition Matrix

$$\boxed{X(s) = \phi(s)\,X(0) + \phi(s)\,B\,U(s)}$$

$$y(t) = C\,x(t)$$

$$Y(s) = C\,X(s) = C\left[\phi(s)\,X(0) + \phi(s)\,B\,U(s)\right]$$

$$Y(s) \xrightarrow{\mathcal{L}^{-1}.T} y(t)$$

$\boxed{EX}$

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} x(t) \quad \& \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

find T.F & $y(t)$ for unit step response.

$\boxed{Sol}$

$$T.F = C(sI - A)^{-1} B$$

$$(sI - A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+3)+2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$*(sI-A)^{-1} = \frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$T.F = C\,(sI-A)^{-1}\,B$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{s^2+3s+2}\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2s \end{pmatrix}$$

$$\boxed{T.F = \frac{2s}{s^2+3s+2}}$$

<u>ch-equation</u>

$$|sI-A| = 0$$

$$s^2+3s+2 = 0$$

$$X(s) = \phi(s)\,X(0) + \phi(s)\,B\,U(s)$$

$$\phi(s) = (sI - A)^{-1} = \frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$X(s) = \frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \ldots$$

$$\frac{1}{s^2+3s+2}\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}\underbrace{\frac{1}{s}}_{U(s)}$$

$$\begin{pmatrix} 0 \\ \frac{2}{s} \end{pmatrix}$$

Cause $u(t) = 1$

$$X(s) = \frac{1}{s^2+3s+2}\left[\begin{pmatrix} 1 \\ s \end{pmatrix} + \begin{pmatrix} \frac{2}{s} \\ 2 \end{pmatrix}\right]$$

$$= \frac{1}{s^2+3s+2}\begin{pmatrix} 1+\frac{2}{s} \\ s+2 \end{pmatrix}$$

$$= \frac{1}{(s+1)(s+2)}\begin{pmatrix} \frac{s+2}{s} \\ s+2 \end{pmatrix}$$

$$X(s) = \begin{pmatrix} \frac{1}{s(s+1)} \\ \frac{1}{s+1} \end{pmatrix}$$

$$y(s) = C\, X(s)$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{s(s+1)} \\ \frac{1}{s+1} \end{pmatrix} = \frac{1}{s+1}$$

$\mathcal{L}^{-1}.T$

$y(t) = e^{-t}$ (unit-step response)

$\boxed{4}$ Controllability

The system is completly controllable if the system states can be changed by changing the system input.

* The ability of the control i/p signal of system to move any initial state to another final states during finite intervals of time

$$x(t_0) \longrightarrow x(t_1)$$

* Controbability matrix (M_c)

$$M_c = (B \quad AB \quad A^2B \cdots A^{n-1}B)$$

على حسب درجة ال system .

<u>if</u> $|M_c| \neq 0$, the system is Controllable.

<u>2nd order</u>

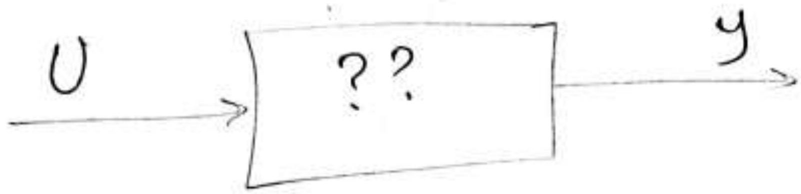
$$M_c = (B \quad AB)$$

<u>3rd order</u>

$$M_c = (B \quad AB \quad A^2B)$$

5 observability

← لو عندك (states) مش عارف تعلمها (estimation)
تستخدم ال (observer)



← لو عارف الخرج وعايز تعرف ما بداخل النظام.
في الحالة دي تقول على النظام إنه (observable)

* In some cases, the states couldn't be measured for the following reasons:
  1- the location for Physical states.
  2- The measuring instruments are not valid.

→ in these case, estimation for these states is required.

* if the internal states of a system Could be estimated (calculated) from the observation of o/p response (o/p) , then system is called observable.

observability Matrix (M_o)

$$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix} \quad ; |M_o| \neq 0 \quad \text{observable}$$

2nd order

$$M_o = \begin{pmatrix} C \\ CA \end{pmatrix}$$

3rd order

$$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$$

$\boxed{EX}$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$$

check observability, Controbability

$$M_c = \begin{pmatrix} B & AB & A^2B \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \quad A^2B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$M_c = \begin{pmatrix} +0 & 1 & 2 \\ -1 & 1 & 0 \\ +0 & 1 & 2 \end{pmatrix}$$

$$|M_c| = - \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$\longrightarrow$ system is not controllable

$$\mathcal{M}_o = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$$

$$CA = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$$

$$CA^2 = CA * A = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \end{pmatrix}$$

$$\mathcal{M}_o = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 4 & -1 \end{pmatrix} \Rightarrow |\mathcal{M}_o| = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= -6 + 2 = -4 \neq 0$$

$\longrightarrow$ system is observable.